Chapter 1

Number Sense

1.1 Numbers

INTEGERS $\mathbb{Z} = \{ \cdots, -3, -2, -1, 0, 1, 2, 3, \cdots \}.$

NEGATIVE INTEGERS $\mathbb{Z}^- = \{\cdots, -3, -2, -1, 0\}.$

POSITIVE INTEGERS $\mathbb{Z}^+ = \{1, 2, 3, \cdots\}.$

WHOLE NUMBERS $\mathbb{Z}^+ \cup \{0\} = \{0, 1, 2, 3, \cdots\}$

NATURAL NUMBERS Same as positive integers. Also known as counting numbers.

REMARK Zero is neither positive nor negative. It is non-negative and non-positive.

FRACTIONS $\frac{a}{b}$, where *a* is known as *numerator* and *b* as *denominator*. $b \neq 0$ and a < b. We use fractions to model partial amounts (*a*) of a whole (*b*). A fraction also represents a ratio or proportion of two quantities. In that case, it is written as a : b.

Example 1.1.1

 $\frac{1}{3}$ and $\frac{3}{4}$ are fractions. The first one may model 1 part of 3 parts ("*the whole*") and the second one 3 out of 4 parts ("*the whole*").

IMPROPER FRACTIONS Fractions are naturally are between 0 and 1. If a given ratio is greater than one it can be represented by an *improper fraction*. There is also a special notation to model amounts where we have a fraction and one or more whole amount. This is called mixed fraction representation. $\frac{a}{b} = p + \frac{q}{b} = p\frac{q}{b}$, where 0 < b < a, q < b, and a = pb + q.

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1.1. NUMBERS | AMC 8 Basic Algebra Chapter I: Number Sense

Example 1.1.2 $\frac{4}{3} = 1 + \frac{1}{3}$ and may also be written as $1\frac{1}{3}$.

REMARK Mixed fractions is best avoided during any mathematical manipulation. You can always put them into and convert from $p + \frac{q}{b} = \frac{a}{b}$ form.

RATIONAL NUMBERS $\mathbb{Q}: \frac{a}{b}$ where both a and b are integers and $b \neq 0$.

Example 1.1.3 $\frac{-9}{97}, \frac{300}{1}$ are rational numbers.

THEOREM All integers are rational numbers. *Why?*

DECIMAL NUMBERS Decimals are not independent set of numbers. They are representation of rational numbers that were invented for computational needs. We find them practical *because we can easily do math with multiples of 10*. You can find decimal equivalent of a rational number by one of many division algorithms. Equivalent of a decimal may be easily written if one considers the *place value* of all the digits.

Example 1.1.4

Convert to decimal representation.

$$\frac{1}{2} = \frac{5}{10} = 5 \cdot 0.1 = 0.5$$
$$\frac{103}{100} = 1 \cdot 100 + 3 \cdot 1 + 3 \cdot 0.01 = 103.03$$

Example 1.1.5 Convert to rational number.

$$0.32 = 3 \cdot 0.1 + 2 \cdot 0.01 = \frac{3}{10} + \frac{2}{100} = \frac{32}{100}$$

PERCENTAGES Just like decimals percentages also exist for practical purposes. Mathematically a percentage is equivalent to a rational number with a denominator of 100. One can expand or simplify to get this denominator. So every rational number can be written as a percentage and vice versa.

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Example 1.1.6

Convert to a percentage.

$$\frac{1}{2} = \frac{50}{100} = 50\%$$
$$0.03 = \frac{3}{100} = 3\%$$

REMARK Multiplication by a fraction is equivalent of finding fraction of that number. So one-third of 9 is $\frac{1}{3} \cdot 9 = \frac{9}{3} = 3$. Since percentages may be written as rational numbers, the same reasoning applies to percentages as well. As an example, fifty-percent of 24 is $50\% \cdot 24 = \frac{50}{100} \cdot 24 = \frac{1}{2} \cdot 24 = 12$.

1.2 **Operations**

ARITHMETICS You probably already know addition, subtraction, multiplication, and division with integers and rational numbers.

Example 1.2.1

$$0 - 14 = -14$$

$$13 - 19 = -6$$

$$-5 - 4 = -9$$

$$-2 \cdot 9 = -18$$

$$-14 \div 2 = -7$$

COMMUTATIVITY When adding numbers a + b is equal to b + a. This is true for multiplication as well: $a \cdot b = b \cdot a$. This property is known as *commutativity*. Subtraction and division operations are not commutative.

REMARK The multiplication operation has two notations: \times and \cdot . Since \times may be confused with variable x, it is best avoided. the division operation has also two notations \div and /.

Example 1.2.2

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$$\begin{aligned} &38,000+42,000=42,000+38,000\\ &7\cdot 5=5\times 7\\ &8\div 2\neq 2/8\\ &4-10\neq 10-4 \end{aligned}$$

ASSOCIATIVITY When adding three numbers (a + b) + c is equal to a + (b + c). This is true for multiplication as well: $(a \cdot b) \cdot c = (a \cdot b) \cdot c$. This property is known as *associativity*. Subtraction and division operations are not associative.

Example 1.2.3

$$(38,997 + 41,000) + 3 = 41,000 + (38,997 + 3)$$

(7 \cdot 5) \cdot 0 = 7 \cdot (5 \cdot 0)
(8/2)/4 \neq 8/(2/4)
(4 - 10) - 6 \neq 4 - (10 - 6)

PARENTHESES Because of associativity and commutativity properties are different for the four arithmetic operations, it is required to use parentheses for clarification purposes in cases where there are more than one way to carry out the operations.

Example 1.2.4

$$(8/2)/4 = \frac{8/2}{4} = \frac{4}{4} = 1$$
$$8/(2/4) = \frac{8}{1/2} = \frac{16}{1} = 16$$

EXPONENTS For two numbers a and n, we define n^{th} power of $a, a^n = a \cdot a \cdots a$. a is called base and n is called the exponent. For n = 2 and n = 3, they have special names: square and cube.

PROPERTIES OF EXPONENTS

- $\diamond a^n = 1$, for all *n* except a = 0. 0^0 is undefined, like 0/0.
- $\diamond a^1 = 1$, for all a.
- $\diamond \ a^m \cdot a^n = a^{m+n}$

♦
$$\frac{a^m}{a^n} = a^{m-n}$$
. A special case is $\frac{1}{a^n} = a^{-n}$, with $m = 0$. This also means that $\frac{1}{a} = a^{-1}$, with $n = 1$.
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- $\diamond \ (ab)^n = a^n \cdot b^n$
- $\diamond \ (a^m)^n = a^{m \cdot n}$

Example 1.2.5

$$(2^3)^2 = 8^2 = 64$$

 $2^{(3^2)} = 2^9 = 512$

REMARK As you can see $(a^m)^n$ is not equal to $a^{(m^n)}$. So a^{m^n} is ambiguous without a convention.

CUSTOM OPERATIONS We can define new operations by using these five basic operations. Since we don't have any conventions established for any new operations, we need to use parentheses to avoid ambiguity.

Example 1.2.6

 $a \spadesuit b$ is defined as $a + b + a \cdot b$.

$$3 (2) = 3$$

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1.3 ORDER OF OPERATIONS

Mathematicians has a convention for the order of well known operations and they don't use parentheses unless it is necessary. A popular mnemonics for the order of operations is "Please Excuse My Dear Aunt Sally" (or PEMDAS: parentheses, exponents, multiplication, division, addition, subtraction). It is also understood that you go from left to right when performing the operations and top to bottom for exponents.

Example 1.3.1

Parenthesize given expressions.

$$1 - 2 + 3 \times 4 \div 4 - 3 + 2 \times 0 = ((((1 - 2) + ((3 \times 4) \div 4)) - 3) + (2 \times 0)))$$

= ((((-1) + (12 \dots 4)) - 3) + 0) = (((((-1) + (3)) - 3) + 0))
= ((2 - 3) + 0) = -1 + 0 = -1

If subtraction a - b is converted to addition a + (-b) then you can use the associativity and commutativity of addition to save many steps.

Example 1.3.2

This time we don't need that many parentheses as addition is both associative and commutative. We can add in any order.

$$1 - 2 + 3 \times 4 \div 4 - 3 + 2 \times 0 = 1 + (-2) + ((3 \times 4)/4) + (-3) + (2 \times 0)$$
$$= (-1) + 3 + (-3) = -1$$

Example 1.3.3

$$2^{3^2} = 2^9 = 512$$

1 - 2 \cdot 3 \dec 4^0 = 1 - 2 \cdot 3 = 1 - 6 = -5

1.4 Exercises

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1. Albus Dumbledore was 51 years old when he first became the Headmaster of Hogwards. If he has been the headmaster for 43 years, how old is he right now?

(A) 92 (B) 93 (C) 94 (D) 97 (E) 99

2. (AMC 8 1999/1) (6?3) + 4 - (2 - 1) = 5. To make this statement true, the question mark between the 6 and the 3 should be replaced by

(A) \div (B) \times (C) + (D) – (E) None of these

3. (AMC 8 1999/3) Which triplet of numbers has a sum NOT equal to 1?

(A)
$$(1/2, 1/3, 1/6)$$
 (B) $(2, -2, 1)$ (C) $(0.1, 0.3, 0.6)$ (D) $(1.1, -2.1, 1.0)$ (E) $(-3/2, -5/2, 5)$

4. (AJHSME 1992/1)
$$\frac{10 - 9 + 8 - 7 + 6 - 5 + 4 - 3 + 2 - 1}{1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + 9} = (A) - 1$$
 (B) 1 (C) 5 (D) 9 (E) 10

5. (AJHSME 1993/1) Which pair of numbers does NOT have a product equal to 36?

(A) $\{-4, -9\}$ (B) $\{-3, -12\}$ (C) $\{\frac{1}{2}, -72\}$ (D) $\{1, 36\}$ (E) $\{\frac{3}{2}, 24\}$

6. (AMC-8 2009/1) Bridget bought a bag of apples at the grocery store. She gave half of the apples to Ann. Then she gave Cassie 3 apples, keeping 4 apples for herself. How many apples did Bridget buy?

(A) 3 (B) 4 (C) 7 (D) 11 (E) 14

7. (AJHSME 1993/4) $1000 \times 1993 \times 0.1993 \times 10 =$

0		2		2
(A) 1.993×10^{3}	(B) 1993 1993	$(\mathbf{C}) (199.3)^2$	(D) 1 993 001 993	(E) $(1993)^2$
(11) 1.000 \land 10	(D) 1000.1000	(C)(100.0)	(D) 1,000,001.000	$(\mathbf{L})(\mathbf{L})(\mathbf{U})$

8. (AJHSME 1989/5) $-15 + 9 \times (6 \div 3) =$ (A) -48 (B) -12 (C) -3 (D) 3 (E) 12

9. (AJHSME 1989/12) $\frac{1-\frac{1}{3}}{1-\frac{1}{2}} =$ (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{3}{2}$ (E) $\frac{4}{3}$

10. (AJHSME 1989/13) $\frac{9}{7 \times 53} =$ (A) $\frac{.9}{.7 \times 53}$ (B) $\frac{.9}{.7 \times .53}$ (C) $\frac{.9}{.7 \times 5.3}$ (D) $\frac{.9}{7 \times .53}$ (E) $\frac{.09}{.07 \times .53}$

11. (AJHSME 1990/6) Which of these five numbers is the largest?

(A) $13579 + \frac{1}{2468}$ (B) $13579 - \frac{1}{2468}$ (C) $13579 \times \frac{1}{2468}$ (D) $13579 \div \frac{1}{2468}$ (E) 13579.2468

12. (AJHSME 1989/8) $(2 \times 3 \times 4) \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) =$ (A) 1 (B) 3 (C) 9 (D) 24 (E) 26

13. (AJHSME 1986/11) If A * B means $\frac{A+B}{2}$, then (3 * 5) * 8 is (A) 6 (B) 8 (C) 12 (D) 16 (E) 30

14. (AMC-8 2014/3) Isabella had a week to read a book for a school assignment. She read an average of 36 pages per day for the first three days and an average of 44 pages per day for the next three days. She then finished the book by reading 10 pages on the last day. How many pages were in the book?

(A) 240 (B) 250 (C) 260 (D) 270 (E) 280

15. What is $7 \cdot (4+2) - 3^2$? What about $7 \cdot 4 + 2 - 3^2$? (A) 33 and 21 (B) 21 and 33 (C) 31 and 23 (D) 23 and 31 (E) 42 and 21

16. For integers a and b, define $a \clubsuit b = 5a - 2b$. What is $(3 \clubsuit 7) \clubsuit (-2)$?

(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

17. Evaluate

$$(72 \div 6) - (70 \div (10 + 25) + 12)$$

(A) - 32 (B) - 2 (C) 2 (D) - 3 (E) 12

18. (AMC 8 2017/1) Which of the following values is largest?

(A) 2 + 0 + 1 + 7 (B) $2 \times 0 + 1 + 7$ (C) $2 + 0 \times 1 + 7$ (D) $2 + 0 + 1 \times 7$ (E) $2 \times 0 \times 1 \times 7$

19. (AMC-8 2016/1) The longest professional tennis match ever played lasted a total of 11 hours and 5 minutes. How many minutes was this?

(A) 605 (B) 655 (C) 665 (D) 1005 (E) 1105

20. If we multiply a number by 3 and subtract it from 40, we get 1. What is the number?

(A) 10 (B) 11 (C) 12 (D) 13 (E) 14

21. (CEMC-Gauss 2012/12) If snow falls at a rate of 1 mm every 6 minutes, then how many hours will it take for 1 m of snow to fall?

(A) 33 (B) 60 (C) 26 (D) 10 (E) 100

22. For rational number x, the function g is defined to be

$$g(x)=\frac{5x+3}{3x+5}$$

Find g(5).

(A) $\frac{28}{10}$ (B) $\frac{14}{20}$ (C) $\frac{7}{5}$ (D) 0 (E) 3

23. (2016 AMC 8/10) Suppose that a * b means 3a - b. What is the value of x if

$$2*(5*x) = 1$$

(A) $\frac{1}{10}$ **(B)** 2 **(C)** $\frac{10}{3}$ **(D)** 10 **(E)** 14

24. (CEMC-Pascal 2014/13) In the subtraction shown, X and Y are digits. What is the value of X + Y?

$$\frac{1X2}{-8Y}$$

$$\frac{-8Y}{45}$$

(A) 15 (B) 12 (C) 10 (D) 13 (E) 9

25. (AMC-8 2017/5) What is the value of the expression $\frac{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8}{1+2+3+4+5+6+7+8}$? (A) 1020 (B) 1120 (C) 1220 (D) 2240 (E) 3360

26. (AMC-8 2017/4) When 0.000315 is multiplied by 7, 928, 564 the product is closest to which of the following?
(A) 210 (B) 240 (C) 2100 (D) 2400 (E) 24000

27. (AMC 8 2011/1) Margie bought 3 apples at a cost of 50 cents per apple. She paid with a 5-dollar bill. How much change did Margie recieve?

(A) \$1.50 (B) \$2.00 (C) \$2.50 (D) \$3.00 (E) \$3.50

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28. (AMC-8 2013/4) Eight friends ate at a restaurant and agreed to share the bill equally. Because Judi forgot her money, each of her seven friends paid an extra \$2.50 to cover her portion of the total bill. What was the total bill?

(A) \$120 **(B)** \$128 **(C)** \$140 **(D)** \$144 **(E)** \$160

29. (AMC-8 2014/7) There are four more girls than boys in Ms. Raub's class of 28 students. What is the ratio of number of girls to the number of boys in her class?

(A) 3:4 (B) 4:3 (C) 3:2 (D) 7:4 (E) 2:1

30. (AMC-8 2012/1) Rachelle uses 3 pounds of meat to make 8 hamburgers for her family. How many pounds of meat does she need to make 24 hamburgers for a neighborhood picnic?

(A) 6 (B) $6\frac{2}{3}$ (C) $7\frac{1}{2}$ (D) 8 (E) 9

31. (AMC-8 2012/2) In the country of East Westmore, statisticians estimate there is a baby born every 8 hours and a death every day. To the nearest hundred, how many people are added to the population of East Westmore each year?
(A) 600 (B) 700 (C) 800 (D) 900 (E) 1000

32. (AMC-8 2012/4) Peter's family ordered a 12-slice pizza for dinner. Peter ate one slice and shared another slice equally with his brother Paul. What fraction of the pizza did Peter eat?

(A) $\frac{1}{24}$ (B) $\frac{1}{12}$ (C) $\frac{1}{8}$ (D) $\frac{1}{6}$ (E) $\frac{1}{4}$

33. (AMC-8 2014/10) The first AMC 8 was given in 1985 and it has been given annually since that time. Samantha turned 12 years old the year that she took the seventh AMC 8. In what year was Samantha born?

(A) 1979 (B) 1980 (C) 1981 (D) 1982 (E) 1983

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34. (AMC-8 2010/11) The top of one tree is 16 feet higher than the top of another tree. The heights of the two trees are in the ratio 3 : 4. In feet, how tall is the taller tree?

(A) 48 (B) 64 (C) 80 (D) 96 (E) 112

35. (AMC-8 2009/2) On average, for every 4 sports cars sold at the local dealership, 7 sedans are sold. The dealership predicts that it will sell 28 sports cars next month. How many sedans does it expect to sell?

(A) 7 (B) 32 (C) 35 (D) 49 (E) 112

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36. (AMC-8 2009/15) A recipe that makes 5 servings of hot chocolate requires 2 squares of chocolate, $\frac{1}{4}$ cup sugar, 1 cup water and 4 cups milk. Jordan has 5 squares of chocolate, 2 cups of sugar, lots of water, and 7 cups of milk. If she maintains the same ratio of ingredients, what is the greatest number of servings of hot chocolate she can make?

(A) $5\frac{1}{8}$ (B) $6\frac{1}{4}$ (C) $7\frac{1}{2}$ (D) $8\frac{3}{4}$ (E) $9\frac{7}{8}$

37. (AMC-8 2008/7) If $\frac{3}{5} = \frac{M}{45} = \frac{60}{N}$, what is M + N? (A) 27 (B) 29 (C) 45 (D) 105 (E) 127

38. (AMC-8 2004/16) Two 600 mL pitchers contain orange juice. One pitcher is 1/3 full and the other pitcher is 2/5 full. Water is added to fill each pitcher completely, then both pitchers are poured into one large container. What fraction of the mixture in the large container is orange juice?

(A) $\frac{1}{8}$ (B) $\frac{3}{16}$ (C) $\frac{11}{30}$ (D) $\frac{11}{19}$ (E) $\frac{11}{15}$

39. (AMC-8 2005/2) Karl bought five folders from Pay-A-Lot at a cost of 2.50\$ each. Pay-A-Lot had a 20%-off sale the following day. How much could Karl have saved on the purchase by waiting a day?

(A) \$1.00 (B) \$2.00 (C) \$2.50 (D) \$2.75 (E) \$5.00

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40. (AMC-8 2013/2) A sign at the fish market says, "50% off, today only: half-pound packages for just \$3 per package." What is the regular price for a full pound of fish, in dollars?

(A) 6 (B) 9 (C) 10 (D) 12 (E) 15

41. (AMC-8 2003/3) A burger at Ricky C's weighs 120 grams, of which 30 grams are filler. What percent of the burger is not filler?

(A) 60% (B) 65% (C) 70% (D) 75% (E) 90%

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42. (AMC-8 2003/5) If 20% of a number is 12, what is 30% of the same number?

(A) 15 (B) 18 (C) 20 (D) 24 (E) 30

43. (AMC-8 2007/6) The average cost of a long-distance call in the USA in 1985 was 41 cents per minute, and the average cost of a long-distance call in the USA in 2005 was 7 cents per minute. Find the approximate percent decrease in the cost per minute of a long- distance call.

(A) 7 (B) 17 (C) 34 (D) 41 (E) 80

44. (AMC-8 2004/6) After Sally takes 20 shots, she has made 55% of her shots. After she takes 5 more shots, she raises her percentage to 56%. How many of the last 5 shots did she make?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

45. (AMC-8 2011/7) Each of the following four large congruent squares is subdivided into combinations of congruent triangles or rectangles and is partially bolded. What percent of the total area is partially bolded?



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(A) $12\frac{1}{2}$

46. (AMC-8 2004/7) An athlete's target heart rate, in beats per minute, is 80% of the theoretical maximum heart rate. The maximum heart rate is found by subtracting the athlete's age, in years, from 220. To the nearest whole number, what is the target heart rate of an athlete who is 26 years old?

(A) 134 (B) 155 (C) 176 (D) 194 (E) 243

47. (AMC-8 2012/8) A shop advertises everything is "half price in today's sale." In addition, a coupon gives a 20% discount on sale prices. Using the coupon, the price today represents what percentage off the original price?

(A) 10 (B) 33 (C) 40 (D) 60 (E) 70

48. (AMC-8 2009/8) The length of a rectangle is increased by 10% percent and the width is decreased by 10% percent. What percent of the old area is the new area?

(A) 90 (B) 99 (C) 100 (D) 101 (E) 110

49. (AMC-8 2008/9) In 2005 Tycoon Tammy invested 100 dollars for two years. During the first year her investment suffered a 15% loss, but during the second year the remaining investment showed a 20% gain. Over the two-year period, what was the change in Tammy's investment?

(A) 5% loss (B) 2% loss (C) 1% gain (D) 2% gain (E) 5% gain

50. (AMC-8 2010/9) Ryan got 80% of the problems correct on a 25-problem test, 90% on a 40-problem test, and 70% on a 10-problem test. What percent of all the problems did Ryan answer correctly?

(A) 64 (B) 75 (C) 80 (D) 84 (E) 86

51. (AMC-8 2005/11) The sales tax rate in Bergville is 6%. During a sale at the Bergville Coat Closet, the price of a coat is discounted 20% from its \$90.00 price. Two clerks, Jack and Jill, calculate the bill independently. Jack rings up \$90.00 and adds 6% sales tax, then subtracts 20% from this total. Jill rings up \$90.00, subtracts 20% of the price, then adds 6% of the discounted price for sales tax. What is Jack's total minus Jill's total?

(A) - \$1.06 (B) - \$0.53 (C) \$0 (D) \$0.53 (E) \$1.06

52. (AMC-8 2003/11) Business is a little slow at Lou's Fine Shoes, so Lou decides to have a sale. On Friday, Lou increases all of Thursday's prices by 10 percent. Over the weekend, Lou advertises the sale: "Ten percent off the listed price. Sale starts Monday." How much does a pair of shoes cost on Monday that cost 40 dollars on Thursday?

(A) 36 (B) 39.60 (C) 40 (D) 40.40 (E) 44

53. (AMC-8 2010/12) Of the 500 balls in a large bag, 80% are red and the rest are blue. How many of the red balls must be removed so that 75% of the remaining balls are red?

(A) 25 (B) 50 (C) 75 (D) 100 (E) 150

54. (AMC-8 2013/12) At the 2013 Winnebago County Fair a vendor is offering a "fair special" on sandals. If you buy one pair of sandals at the regular price of \$50, you get a second pair at a 40% discount, and a third pair at half the regular price. Javier took advantage of the "fair special" to buy three pairs of sandals. What percentage of the \$150 regular price did he save?

(A) 25 (B) 30 (C) 33 (D) 40 (E) 45

55. (AMC-8 2006/12) Antonette gets 70% on a 10-problem test, 80% on a 20-problem test and 90% on a 30-problem test. If the three tests are combined into one 60-problem test, which percent is closest to her overall score?

(A) 40 (B) 77 (C) 80 (D) 83 (E) 87

56. (AMC-8 2002/14) A merchant offers a large group of items at 30% off. Later, the merchant takes 20% off these sale prices and claims that the final price of these items is 50% off the original price. The total discount is

(A) 35% (B) 44% (C) 50% (D) 56% (E) 60%

57. (AMC-8 2017/14) Chloe and Zoe are both students in Ms. Demeanor's math class. Last night they each solved half of the problems in their homework assignment alone and then solved the other half together. Chloe had correct answers to only 80% of the problems she solved alone, but overall 88% of her answers were correct. Zoe had correct answers to 90% of the problems she solved alone. What was Zoe's overall percentage of correct answers?

(A) 89 (B) 92 (C) 93 (D) 96 (E) 98